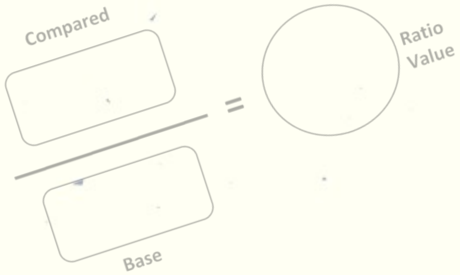

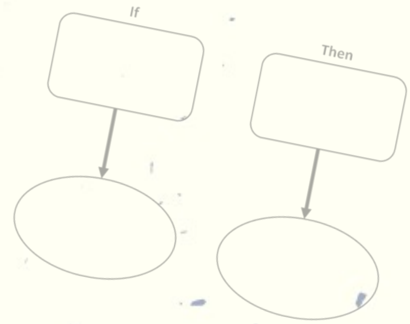

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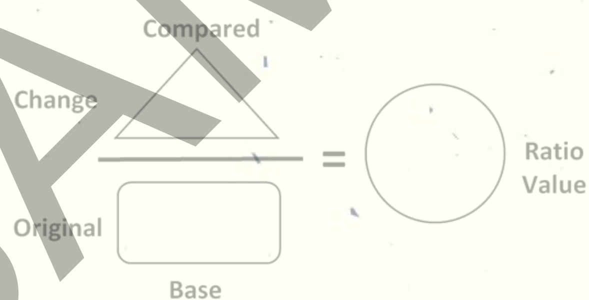
SOLVING RATIO, PROPORTION, & PERCENT PROBLEMS USING SCHEMA-BASED INSTRUCTION

Program Sampler



$\frac{x}{50} = \frac{65}{100}$

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The examples in this curriculum are used in a fictitious manner. We are not making statements about any specific person, character, business, place, or event.

Schema Based Instruction (SBI) Curriculum

Scope and Sequence

LESSON	CONTENT	SCHEDULE
	Unit 1: Ratios and Proportions	
1 & 2	Ratios	3 days
3 & 4	Solving Ratio Word Problems: Part-to-Part and Part-to-Whole Comparisons	3 days
5	Rates (Quiz over Lessons 1-4)	1 day
6 & 7	Solving Proportion Problems	3 days
8 & 9	Scale Drawing Problems (Quiz at end of Lesson 9 over Lessons 1-7)	3 days
10	Unit 1 Review	1 day

LESSON	CONTENT	SCHEDULE
	Unit 2: Percents	
11	Fractions, Percents, & Decimals	3 days
12	Solving Percent Problems: Part-to-Whole Comparisons	
13 & 14	Solving Percent Problems: Percent of Change	3 days
15	Solving Sales Tax & Tips Problems: Percent of Change (Quiz over Lessons 11-14)	3 days
16	Solving Markup & Discount Problems: Percent of Change	
17	Solving Multistep Percentage Adjustment Problems	3 days
18	Solving Simple Interest Problems	
19	Review: Identifying and Categorizing Problem types (Ratio, Proportion, and Percent)	1 day
20	Unit 2 Review	1 day
21	Units 1 & 2 Review	1 day
	Total	**29 days**

Note. Schedule is based on 45-60 minute class period.

Guide to Teacher Materials

For each lesson in the teacher guide, there are corresponding materials to support instruction.

Teacher Guide: This includes scripted materials for teaching each lesson. The scripts are not intended to be read verbatim, but rather to be reviewed and to guide you in developing students' proportional reasoning using the specified examples and problem solving activities. Worked answers for the Practice Problems for each lesson are included at the end of the corresponding lesson.

Lessons may also include the following:

- Detailed instructions or procedures (e.g., Think-Plan-Share)
- DISC checklists
- Jeopardy PowerPoint (Lesson 19)

There are quizzes and quiz answer keys in the following lessons:

- Lesson 5
- Lesson 9
- Lesson 15

Corresponding Materials:

- **Student Workbook**: This includes blank copies of all problems included in the Teacher Guide. This book also includes a reference guide for solving each problem type, located throughout the corresponding lessons. A DISC checklist is included at the end of the book for students to refer to when solving problems.

- **Student Homework Book**: This includes blank copies of all problems included in the Teacher Homework Answer Key. This book also includes a reference guide for solving each problem type, located at the end of the book. A DISC checklist is included at the end of the book for students to refer to when solving problems.

- **Teacher Homework Answer Key**: These are the answers (with explanations) for the student homework book. The first page of each lesson in this booklet is an abbreviated answer key with only the answers.

Lesson 1: Ratios

Lesson Objectives

Students define ratio as a multiplicative relationship. They identify the base quantity for comparison of quantities involving part-to-part and part-to-whole.

Vocabulary: Compared quantity, base quantity, front term, back term, ratio, value of the ratio, part-to-part ratio, part-to-whole ratio

CCSSM: 6.RP.A.1, 6.RP.A.3, 7.NS.A.2, 7.NS.A.2.B, 7.RP.A.2, 7.RP.A.2.A

Lesson 1 Overview

Introduction:

- Review equivalent fractions and related word problems.
- Review simplifying fractions.
- Define ratio as a multiplicative relationship. A ratio is a comparison of any two quantities or measures, and expresses a multiplicative relationship between two quantities.
- Introduce part-to-part and part-to-whole ratios.

Example 1.6: Discuss ratios and allow for student thinking.

- Compare the height of the two trees by examining compared quantity and base quantity.

Example 1.7: Students work on tasks individually and then as a class to discover the meaning of ratios. Using the teacher and student desk pictures, students will:

- Represent the relationship between the two quantities as a ratio.
- Write the value of the ratio.
- Label the front and back terms.
- Describe the relationship between the two quantities.

Example 1.8: Use diagrams to represent the ratios.

- Compare the number of brownies Lisa and Tim bought.
- Ask students how many times the number of brownies Lisa bought is to the number of brownies Tim bought and vice versa.

Example 1.9: Use think-aloud to model solving the problem.

- Compare the relationship between Ryan and Steve's heights.

Example 1.10: Explain the meaning of ratios in words.

- Discuss survey results of 7^{th} graders' favorite drink (Orangina vs. Sprite).
- Find the ratio and the value of each ratio, and say what the ratio means using words and numbers.
- Check to make sure students understand the meaning of ratios and ratio values.

Challenge Problems (optional):

Challenge Problem 1.11: Have students write two ratios comparing the time that Lucinda spent on homework on Monday to the time she did homework on Wednesday.

Lesson 1: Ratios

- Remind students to convert the two different units of time (minutes and hours) to a common unit of time.

Challenge Problem 1.12: Have students solve for Stacy's weight on the moon.

- Remind students to consider how the two ratios/rates are similar (astronaut: $\frac{174 \text{ lbs on Earth}}{29 \text{ lbs on the moon}}$; Stacy: $\frac{102 \text{ lbs on Earth}}{x \text{ lbs on the moon}}$)

Warm-Up/Review of Equivalent Fractions

Teacher: 1.1 Circle the equivalent fractions (there are 3 equivalent fractions in each set):

$$\frac{1}{2} \quad \frac{5}{10} \quad \frac{7}{15} \quad \frac{6}{12} \qquad\qquad \frac{3}{9} \quad \frac{2}{8} \quad \frac{6}{18} \quad \frac{1}{3}$$

1.2 Indicate whether each pair of fractions is equivalent or not (Yes or No):

$\frac{3}{5}, \frac{12}{20}$ (Yes or No) $\qquad$ $\frac{25}{40}, \frac{30}{45}$ (Yes or No)

1.3 Complete the equivalent fraction by filling in the missing part:

$$\frac{1}{2} = \frac{3}{\underline{\quad}} \qquad \frac{1}{7} = \frac{\underline{\quad}}{21} \qquad \frac{4}{\underline{\quad}} = \frac{5}{25}$$

1.4 Simplify each fraction:

$$\frac{12}{15} = \qquad \frac{22}{44} = \qquad \frac{6}{14} = \qquad \frac{10}{35} = \qquad \frac{9}{27} =$$

Fraction Word Problem:

1.5 A class is having a pizza party and they ordered 2 pizzas from different restaurants. The class ate $\frac{5}{6}$ of the pizza from Don's Pizzeria and $\frac{9}{12}$ of the pizza from Perfect Pizza. Did they eat the same amount of pizza from each restaurant? Explain your reasoning.

Teaching the Lesson

Ratio as a Multiplicative Relationship

Teacher: Today we will learn to compare quantities.

(Display Example 1.6)

Example 1.6	There are two trees in this picture. One is 6 feet tall and the other is 2 feet tall. There are many ways that you might compare the heights of these two trees. Can someone tell me one way that you can compare these two quantities?

6 ft. tree

2 ft. tree

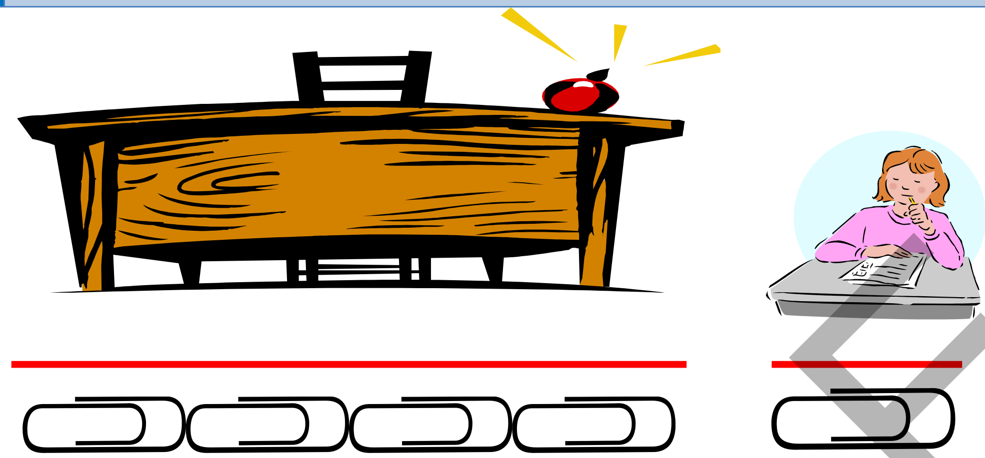
Discussion points: *Give students a few moments to think about this question by themselves and/or to chat with a person nearby. Walk about the room to identify students who discussed the additive relationship between the two quantities. Call on one of these students first to present their response followed by responses that focused on the multiplicative relationship. Note that even though both additive and multiplicative responses will come up and be discussed, optimally the emphasis should be more on the multiplicative responses.*

Possible student response	Teacher notes	Teacher response
The 6-ft tree is 4 feet taller than the 2-ft tree. ***OR*** *6 ft minus 2 ft is 4 feet.*	*This response focuses on the* ***additive relationship*** *between the two quantities.*	Excellent! So one/another way we can represent the relationship between the two trees is as follows *(write on board)*: **The 6-ft tree is 4 feet taller than the 2-ft tree.** This tells us how much taller the 6-ft tree (**compared quantity**) is than the 2-ft tree (**base quantity**). This relationship describes the difference (6 - 2 = 4) between the two quantities, and the difference is the number of feet you would add to the 2-ft tree to reach the height of the 6-ft tree.
I measured the trees, and the 6-ft tree is three times the height of the 2-ft tree. ***OR*** *6 ft divided by 2 ft is 3.*	*These responses focus on the multiplicative relationship between the two quantities.* *Students may respond 3:6 or 6:3. Either response is correct.*	Great! So one/another way we can represent the relationship between the two trees is as follows (write on board): The 6-ft tree **(compared quantity)** is 3 times taller than the 2-ft tree **(base quantity)**. This relationship describes a multiplicative relationship (3 times) between the two quantities. We can represent the relationship between the two quantities as a ratio. **A ratio is a comparison of any two quantities or measures. A ratio expresses a multiplicative relationship between two quantities in a single situation.** For example, in the situation involving these two trees, the ratio would be 6 **by** 2, or **6:2** (i.e., six to two). The result of dividing 6 by 2 (6 ÷ 2), 3 (quotient), is called **the value of the ratio 6:2**, where 6 is called the **front term (compared quantity) of the ratio** and 2 is called the **back term (base quantity) of the ratio**. This idea is the same as saying “6 is 3 times as many as 2.” In other words, the value of the ratio 6:2 describes the multiplicative relationship between the 6-ft tree (compared quantity) and the 2-ft tree (base quantity).

Teacher: I have introduced some new words to describe comparisons between quantities. Let’s do another example to practice using these words.

(Display Example 1.7)

Example 1.7	Here is a picture of two desks. When you measure these pictures of the teacher and student desks in paper clips, the teacher desk is 4 paper clips long and the student desk is 1 paper clip long.



Teacher: I want you to look at the pictures *(point to pictures)* of the teacher and student desks and do the following *(point to statements on the teaching transparency)*: **(1) Represent the relationship between the length of the two desks as a ratio (i.e., a comparison that expresses a multiplicative relationship between the two quantities or measures), (2) write the value of the ratio, (3) label the front term (compared quantity) and the back term (base quantity), and (4) describe the relationship between the two quantities in words.**

(Give students a few minutes to complete these 4 tasks, individually. If students are sitting in groups, consider giving the groups time to go over their answers before you go over these with the class. Reinforce that a ratio value is the value found when you divide the front term by the back term).

Possible student response	Teacher notes	Teacher response
	Represent the two quantities as a ratio.	
4:1	*Either response is correct; however, the ratios represent different comparisons, so the ratios are not "the same." If students ask why they are different, at this point, simply respond that they are different because they have different front and back terms.*	Very good! This ratio uses a multiplicative relationship (4:1) to compare the length of the teacher's desk to the length of the student's desk.
1:4		Great - you thought of a different ratio, 1:4. This ratio is different, because it involves a multiplicative comparison of the length of the student's desk to the length of the teacher's desk. Both ratios of 4:1 and 1:4 correctly describe this single situation involving the lengths of these two desks using a multiplicative relationship.

Possible student response	Teacher notes	Teacher response
Write the value of the ratio.		
$4 \div 1 = 4$	*Either response is correct; however, because they are different ratios, they also have different ratio values. The different ratio values provide information about the relationship between the lengths of the two desks.* *Reinforce that a ratio value is the value found when you divide the front term by the back term.*	Perfect! To find the value of the ratio you divide the front term by the back term.
$1 \div 4 = \frac{1}{4}$ or 0.25		Wonderful! You divided the front term by the back term to get the ratio value of $\frac{1}{4}$ or 0.25. **Because the ratio 4:1 is different than the ratio 1:4, you also found that these ratios have different ratio values. In the ratio 4:1, the ratio value of 4 tells you that the teacher's desk is 4 times as long as the student's desk. In the ratio 1:4, the ratio value tells you that the student's desk is $\frac{1}{4}$ the size of the teacher's desk.**
Label the front term (compared quantity) and the back term (base quantity).		
4 is the front term (compared quantity) and 1 is the back term (base quantity).		Yes! For the ratio 4:1, 4 is the front term (compared quantity) and 1 is the back term (base quantity).
1 is the front term (compared quantity) and 4 is the back term (base quantity).		You are doing a great job! Yes, for the ratio 1:4, 1 is the front term (compared quantity) and 4 is the back term (base quantity).
Describe the relationship between the two quantities in words.		
The ratio of the length of the teacher's desk to the length of the student's desk is 4:1.	*All responses are correct; however, the wording of the ratios emphasizes that the order of the numbers in the ratio is very important and a different order will communicate different information about the*	Excellent! This wording emphasizes which desk length is the compared quantity (i.e., teacher's desk) and which desk length is the base quantity (i.e., student's desk).
The length of the teacher's desk is 4 times longer than the length of the student's desk.		Way to go! This wording describes the multiplicative relationship between the lengths of the two desks, and indicates how much longer the teacher's desk (compared quantity) is than the student's desk (base quantity) for these two specific desks.

The ratio of the length of the student's desk to the length of the teacher's desk is 1:4.	*relationship between the desks' lengths.*	You are doing such a good job! This wording identifies the length of the student's desk as the compared quantity and the length of the teacher's desk as the base quantity.
The student's desk is $\frac{1}{4}$ the length of the teacher's desk.		Wonderful! With this wording, you are highlighting the multiplicative relationship between the lengths of the two desks in this situation, which indicates how much smaller the student's desk is than the teacher's desk.

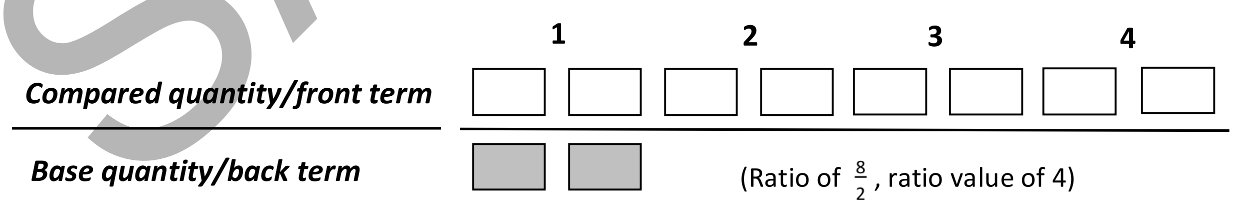
Teacher: **Ratio problems describe a multiplicative relationship between two quantities in a single situation.** For example, you could find the ratio of a 12 oz. (small) water bottle and a 16 oz. (large) water bottle from the vending machine, or you could find the ratio of the number of R rated movies to the number of PG rated movies at the movie theater. In each of these examples, we are talking about one setting, the number of water bottles in a specific vending machine or the number of movies at a specific theater. Let's look at another example, where we are multiplicatively comparing two quantities in one situation.

The Meaning of the Ratio Value

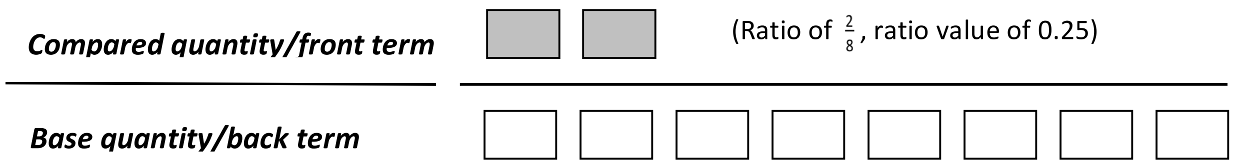
(Display Example 1.8)

Example 1.8	Lisa and her brother, Tim, went to the store today and bought some brownies. Lisa bought 8 brownies and Tim bought 2 brownies. Answer the following questions: (1) What is the ratio of the number of Lisa's brownies to the number of Tim's brownies? (2) The number of brownies Lisa bought is how many times the number of brownies Tim bought? (3) What is the ratio of the number of Tim's brownies to the number of Lisa's brownies? (4) The number of brownies Tim bought is how many times the number of brownies Lisa bought? For all solutions, represent the ratio relationship using a diagram.

Discussion points: *Call on students to share their solutions and representations. Students may not have difficulty solving the problem, but may encounter difficulty representing the ratios. Use the following representation to accompany the discussion of ratios 8:2 and 2:8.*



Teacher: *(Write on board as you discuss the solution.)* The answer to the first question is 8:2, which represents the ratio of the number of Lisa's brownies to the number of Tim's brownies. **The solution to the second question is 8 ÷ 2 = 4. To solve this problem, you needed to identify the base quantity.** Because you are using a multiplicative relationship (8:2) to compare the number of Lisa's brownies **to** the number of Tim's brownies, the number of brownies Tim bought is the base quantity (back term).



The answer to the third question is 2:8, which represents the ratio of the number of Tim's brownies to the number of Lisa's brownies. In this situation, you are multiplicatively comparing the number of brownies Tim bought to the number of brownies that Lisa bought. Therefore, the number of Lisa's brownies is the base quantity (back term) for comparison. The solution to the fourth problem is $2 \div 8 = \frac{2}{8} = 0.25$. **You can use both fractions and decimals for the ratio value. These examples of different ratios (8:2 and 2:8) are within the context of a single situation involving brownies made by Lisa and Tim and show us that a ratio is a multiplicative relationship between two quantities. Also, notice that a ratio is a way to compare two quantities using the division operation. Therefore, the order of the two terms in a particular ratio is important. If you noticed the solutions to the two questions, it is clear that the value of ratio 8:2 (4) is not the same as the value of ratio 2:8 (0.25). Yet both ratio values tell us important information about this specific situation with Lisa and Tim baking brownies.**

(Display Example 1.9)

Example 1.9	Ryan, a basketball player, is 6 ft tall, and his friend Steve, who is shorter than Ryan, is 60 inches tall. Ryan says that the ratio between his height and Steve's height is 6:60. Is 6:60 the best way to describe the relationship between Ryan and Steve's heights?

Discussion points: *Some students may agree and reason that we are comparing 6 (front term or compared quantity, which is Ryan's height) to 60 (back term, or base quantity, which is Steve's height). Model (think-aloud) how you would solve this problem.* ***It is important to model your thinking (as the script below illustrates) rather than tell students how to solve the problem. Also, model thinking that may not be correct and that will require adjusting the strategy, as is done below.*** *In this situation, show students the importance of forming a more meaningful relationship* ***by focusing on the units*** *when comparing two quantities.*

Teacher: 6:60 is one ratio that expresses a multiplicative relationship between Steve's and Ryan's heights. I know that a ratio compares two quantities, where one quantity is the compared quantity and the other is the base quantity. **In this problem, 6 is the compared quantity and 60 is the base quantity, because we are comparing Ryan's height <u>to</u> Steve's height. I can also represent this same ratio as $\frac{6}{60}$. The result of $6 \div 60$ is $\frac{1}{10}$ or 0.1. That is, Ryan is 0.1 times as tall as Steve.** This does not seem to make sense to me, because we know that Ryan is taller than his friend Steve. I know that I correctly used the base quantity for comparison. Let me read the problem and underline Ryan's and Steve's heights. *Ryan, a basketball player, is <u>6 ft</u> tall, and his friend Steve* is *<u>60 inches</u> tall.* It looks like that we are comparing feet to inches. I think it would help if we compared the two heights using the same measure (i.e., feet or inches). **What can I do?**

This problem gives Ryan's height in feet and Steve's height in inches. **One way to make a better ratio would be to convert both heights to inches. Since there are 12 inches in a foot, I can convert 6 feet (Ryan's height) to inches by multiplying how tall he is in feet by 12.** So 6 times 12 is 72, so Ryan is 72 inches tall. **So the ratio using both heights in inches would be 72:60.**

Alternatively, I can **write a better ratio by converting both heights to feet.** Ryan's height is already in feet, so I just need to convert Steve's height. **Steve is 60 inches tall, so I divide 60 by 12 to convert to feet.** 60 divided by 12 is 5, so Steve is 5 feet tall. **So the ratio in feet of Ryan's height to Steve's height is 6:5.** *(If the question arises as to whether these two ratios, with both quantities in feet or both quantities in inches, are the same as each other or the same as the original 6:60 ratio, say that the topic of what makes one ratio the same as another will be talked about in the next lesson. But the point here is that many different ratios can be written for the same situation, and each of these ratios tells us something about the multiplicative relationships that are present in the given quantities.)*

Just to make sure that we are all on the same page, let's return to the situation involving Tim and Lisa's brownies and try to write as many ratios as we can that describe this situation. Remember that Tim had 2 brownies and Lisa had 8 brownies. How many brownies do they have altogether? (10) Now give me examples of different ratios that describe this situation.

Students: *The ratio of the number of Tim's brownies to the number of Lisa's brownies is 2:8. And the ratio of the number of Lisa's brownies to the number of Tim's brownies is 8:2.*

Teacher: What other ratios can you come up with in this situation?

Students: *The ratio of the number of Tim's brownies to the total number of brownies is 2:10. And the ratio of the number of Lisa's brownies to the total number of brownies is 8:10.*

Teacher: As we'll talk about soon, not all of these ratios are equivalent; 2:10 is not equal to 8:2 is not equal to 2:8. Yet all of these ratios tell us information about the relationship between the brownies that Lisa had and the brownies that Tim had. Let's keep exploring this very important idea with another example.

(Display Example 1.10)

Example 1.10

In a survey of 50 seventh graders, 10 chose Orangina as their favorite drink and 40 chose Sprite as their favorite drink. Find the following ratios and the value of each ratio, and then say what the ratio means using words and numbers.

(1) The number of 7th graders who chose Sprite to the number of 7th graders who chose Orangina
(2) The number of 7th graders who chose Sprite to the total number of 7th graders
(3) The number of 7th graders who chose Orangina to the number of 7th graders who chose Sprite
(4) The number of 7th graders who chose Orangina to the total number of 7th graders

(Note: Requiring students to "say what the ratio means using words and numbers" will not be initially clear to them, so you may have to rephrase as follows: "quantity 1 is -- times as large/small as quantity 2.")

Possible student response	Teacher notes	Teacher response
The number of 7th graders who chose Sprite to the number of 7th graders who chose Orangina (ratio, ratio value, and meaning in words)		
***Ratio**: 40:10* ***Value:** 40 ÷ 10 = 4* ***Meaning:** The 40 students who chose Sprite is 4 times as large as the 10 students who chose Orangina.*	*Emphasize that this ratio is a multiplicative comparison of two parts of the seventh grade population – the 40 students who chose Sprite and the 10 students who chose Orangina. **This is called a part-to-part comparison,** because it compares part of a set (the number of students who chose Sprite) to another part (the number of students who chose Orangina) of the same set (the total of all seventh-grade students who were surveyed).*	Yes, this ratio describes two groups of students: 40 seventh graders who chose Sprite compared to 10 seventh graders who chose Orangina. **The ratio value of 4 tells us that the number of seventh graders (compared quantity) who chose Sprite is 4 times the number of students who chose Orangina (base quantity).**
***Ratio:** 10:40* ***Value:** 10 ÷ 40 = $\frac{1}{4}$ or 0.25* ***Meaning:** The 10 students who chose Orangina is $\frac{1}{4}$ as large as the 40 students who chose Sprite.*		This is actually the answer to question #3. In question #1 we are comparing the number of students who chose Sprite (the front term/compared quantity) to the number of students who chose Orangina (the back term/base quantity). Remember, when you set up your ratio, you have to have the correct front and back terms. If not, the ratio value will not be correct. Let's think about this ratio value -- does it make sense that the 40 students who chose Sprite is $\frac{1}{4}$ of the 10 students who choose Orangina when 40 is greater than 10? No, so the ratio value of $\frac{1}{4}$ does not make sense.

Possible student response	Teacher notes	Teacher response
The number of 7th graders who chose Sprite to the total number of 7th graders (ratio, ratio value, and meaning in words)		
***Ratio**: 40:50* ***Value:** $40 \div 50 = \frac{4}{5}$ or 0.8* ***Meaning:** The 40 students who chose Sprite is $\frac{4}{5}$ as large as the 50 students surveyed*	*Emphasize that this ratio is a multiplicative comparison of a part of the seventh-grade population (students who chose Sprite) to **all** of the seventh graders surveyed. **This is called a part-to-whole comparison,** because it compares part of a set (the number of students who chose Sprite) to the whole set (all of the seventh graders surveyed).*	That's right! We are using a multiplicative relationship (40:50) to compare the number of seventh graders who chose Sprite (the front term or compared quantity) to all of the seventh graders surveyed (the back term or base quantity). **This ratio value of $\frac{4}{5}$ or 0.8 tells us that the number of seventh graders who chose Sprite is $\frac{4}{5}$ the total number of seventh graders surveyed.**
***Ratio:** 40:10, 10:40* ***Value**: $40 \div 10 = 4$, $10 \div 40 = \frac{1}{4}$ or 0.25*		With these ratios, we are still using a multiplicative relationship to compare the two parts of the whole group of seventh graders surveyed (students who chose Sprite and students who chose Orangina). The question is asking us to compare one part of the seventh graders (students who chose Sprite) to all of the seventh graders surveyed. Because the ratios are not written in the correct order and/or do not identify the correct quantities, you get a ratio value that does not provide you with the right information to make the comparison.
***Ratio:** 50:40* ***Value:** $50 \div 40 = \frac{5}{4}$ or 1.25*		This ratio uses the right numbers but in the wrong order—we want to compare the students who chose Sprite (front term or compared quantity) to the whole group of seventh graders surveyed (back term or base quantity). Because the ratios are not written in the correct order and/or do not identify the correct quantities, you get a ratio value that does not provide you with the right information to make the comparison.

Possible student response	Teacher notes	Teacher response
The number of 7th graders who chose Orangina to the number of 7th graders who chose Sprite (ratio, ratio value, and meaning in words)		
***Ratio**: 10:40* ***Value**: 10 ÷ 40 =* $\frac{1}{4}$ *or 0.25* ***Meaning:** The 10 students who chose Orangina is* $\frac{1}{4}$ *as large as the 40 students who chose Sprite.*	*Emphasize that this ratio is a multiplicative comparison of two parts of the seventh grade population – the 10 students who chose Orangina and the 40 students who chose Sprite. **This is called a part-to-part comparison.***	Perfect! This ratio describes two groups of students: 10 students who chose Orangina compared to the 40 students who chose Sprite. The ratio value of $\frac{1}{4}$ tells you that the number of students who chose Orangina is $\frac{1}{4}$ times the number of students who chose Sprite.
***Ratio:** 40:10* ***Value:** 40 ÷ 10 = 4* ***Meaning:** The 40 students who chose Sprite is 4 times as large as the 10 students who chose Orangina.*		Remember, you need to be careful when writing the ratio -- this ratio 40:10 compares the number of students who chose Sprite to the number of students who chose Orangina. **The correct ratio uses a multiplicative relationship to compare the number of students who chose Orangina (compared quantity) to the number of students who chose Sprite (base quantity).** Writing the ratio in the wrong order for the comparison gives you the incorrect ratio value. We know that fewer students chose Orangina than Sprite, so the ratio value of 4 times (meaning 4 times as many students chose Orangina as chose Sprite) does not make sense.
The number of 7th graders who chose Orangina to the total number of 7th graders (ratio, ratio value, meaning in words)		
***Ratio:** 10:50* ***Value:** 10 ÷ 50 =* $\frac{1}{5}$ *or 0.2* ***Meaning:** The 10 students who chose Orangina is* $\frac{1}{5}$ *as large as the 50 students surveyed.*	*Emphasize that this ratio is a multiplicative comparison of a part of the seventh grade population (the Orangina-choosing students) to all of the seventh graders surveyed. **This is called a part-to-whole comparison.***	Perfect! This ratio uses a multiplicative relationship to compare a part of the seventh graders (students who chose Orangina) to the whole group of seventh graders surveyed. The ratio value tells you that the number of students who chose Orangina is $\frac{1}{5}$ the total number of seventh graders surveyed.

Possible student response	Teacher notes	Teacher response
Ratio: *10:40, 40:10* ***Value:*** *$40 \div 10 = 4$, $10 \div 40 = \frac{1}{4}$*		With these ratios, you are still using a multiplicative relationship to compare the two kinds of seventh graders to each other, rather than comparing one part of the seventh graders (students who chose Orangina) to all seventh graders surveyed. Because the ratios are not written in the correct order and/or do not identify the correct quantities, you get a ratio value that does not provide you with the right information to make the comparison.
Ratio: *50:10* ***Value:*** *$50 \div 10 = 5$*		This ratio asks you to use a multiplicative relationship (50:10) to compare part of the seventh graders (those who chose Orangina) to all of the seventh graders surveyed. You have the correct ratio terms but in the wrong order. Because the ratios are not written in the correct order and/or do not identify the correct quantities, you get a ratio value that does not provide you with the right information to make the comparison.

Challenge (optional)

(Display Challenge Problem 1.11)

Challenge Problem 1.11	Lucinda has been thinking about how much time she spent on homework last week. She worked on homework for a half hour on Monday and 20 minutes on Wednesday. Compare the time Lucinda spent on homework on Monday to the time she did homework on Wednesday. Write two ratios that compare these two times.

Challenge Problem 1.11: Answer

The amount of time Lucinda spent on homework on Monday to the amount of time she spent on homework on Wednesday:

1. In minutes: 30: 20, which is a ratio of 3:2 (30: 20 is the same as $\frac{30}{20}$, so we can simplify it to 3:2).

2. In hours: $\frac{1}{2} : \frac{1}{3}$

Answer: 3 : 2 in minutes or $\frac{1}{2} : \frac{1}{3}$ in hours.

(Display Challenge Problem 1.12)

Challenge Problem 1.12	An astronaut who weighs 174 lb on Earth weighs 29 lb on the moon. If Stacy weighs 102 lb on Earth, how much would she weigh on the moon?

Challenge Problem 1.12: Answer

Astronaut: $\frac{\text{174 lb on Earth}}{\text{29 lb on the moon}}$	**Stacy:** $\frac{\text{102 lb on Earth}}{x \text{ lb on the moon}}$

$$\frac{\text{174 lb on Earth}}{\text{29 lb on the moon}} = \frac{\text{102 lb on Earth}}{x \text{ lb on the moon}}$$

$$174 \cdot x = 102 \cdot 29$$
$$174x = 2{,}958$$
$$x = 17$$

Answer: 17 lb.

Homework: pp. 1-6

Lesson 19: Review: Identifying and Representing Problem Types

Lesson Objectives

Students learn to identify and categorize word problems into appropriate problem types (e.g. ratio, proportion, percent and percent of change).

Vocabulary: interest, principal, simple interest, annual interest rate, balance, ratio, proportion, percent, percent of change

CCSSM: Standards included in lessons 11-18

Review

(about 10 minutes)

Teacher: Yesterday we solved simple interest problems. What are some situations when we use simple interest?

Students: *You might get simple interest if you deposit money in a bank or you might pay simple interest if you get a loan.*

Teacher: Exactly! Simple interest is the rate you earn on your money or pay for borrowing money. Is the simple interest rate expressed as a decimal, fraction, or percent?

Students: *Percent.*

Teacher: Right. Who remembers what the term is for the amount of money you deposit?

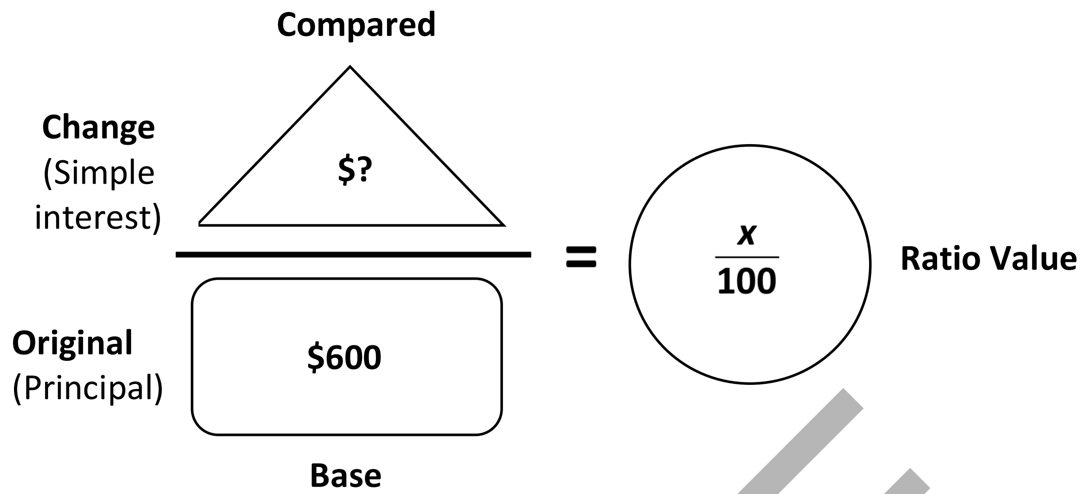
Students: *Principal.*

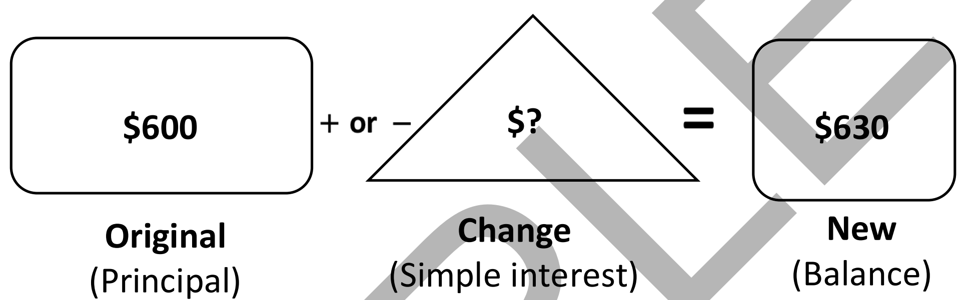
Teacher: Right. When we think about simple interest problems, remember that you may be trying to **find the amount that was first deposited or borrowed, the simple interest rate, the amount earned or paid in interest, or the final balance.** Let's solve this problem.

(Display Review Problem 19.1)

Review Problem 19.1	You deposit \$600 into a certificate of deposit. After 1 year the balance is \$630. Find the simple annual interest rate.

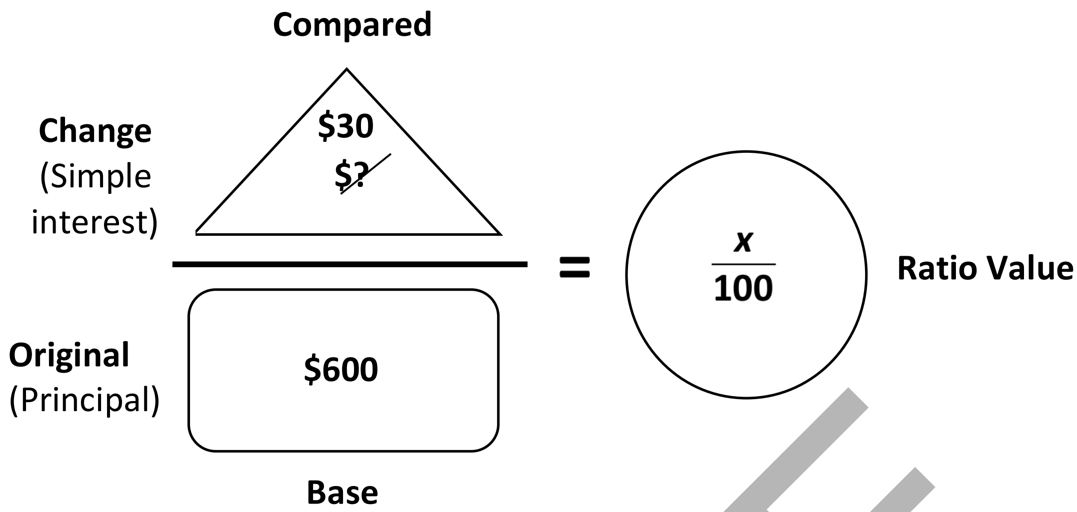
Step 1: *Fill the information given in the problem onto the Ratio and Change diagrams.*

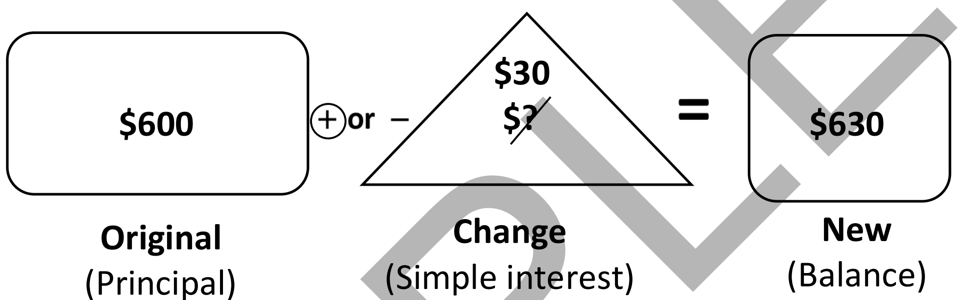




Step 2: Solve first for the "?" (change amount) in the Change diagram. Remember, simple interest is always added to the principal amount. So, circle the "+" in the change diagram and think what number you would add to $600 to get $630 or subtract $630 from $600. $630 — $600 = $30. So, $30 was the amount earned in interest in one year.

Step 3: Cross out the "?" for the change amount in the Change diagram and write in $30 to indicate the amount of interest earned. Also, cross out the "?" for the change amount in the Ratio diagram and write in $30.





Step 4: Solve for the interest rate for 6 months in the Ratio diagram.

$$\frac{\$30}{\$600} = \frac{x}{100}$$

600 ÷ 100 = 6, so 30 ÷ 6 = 5

Teacher: What is the simple interest rate in this problem?

Students: *5%.*

Answer: You would have earned **5% simple interest** on your deposit.

Teaching the Lesson

Unit Review

Teacher: In this unit, we learned about four types of problems. Who can name them?

Students: *Ratio, proportion, percent, and percent of change (including simple interest).*

(Note: You may want to write these problem types on the board so that students can refer to them as the review game progresses. Help students see that all of the problems covered in the

unit fit into these basic categories. For example, if students mention problem types like commission or prediction, point out that these are basically percent problems. If they name markup or discount problems, point out that these are percent of change problems. Tip problems may be either percent or percent of change.)

Well done! Today, we are going to play "Jeopardy!" to practice sorting problems into each of these categories. Sometimes the most difficult part of solving a word problem is figuring out what kind of problem you have. This is something we've been working hard on in this unit and something that the DISC checklist helps with. So we are going to practice sorting with this review activity.

(Note: The Jeopardy game is in a PowerPoint format. The rules will be essentially the same as a generic Jeopardy game. Students should play in teams/groups/ partners for about 15-20 minutes. Each group will produce a response in "Jeopardy Talk" – e.g. "What is a ratio problem?" A whiteboard for each group would be ideal, but notebook or pieces of scratch paper would also work. After the problem is displayed, each group will write an answer indicating what type of problem they see. You can then ask groups to reveal their answers and award points according to the point-value of the question – e.g. "Shopping for 30 points". Groups can keep track of their points or you can appoint a score keeper for the class. It is recommended to play regular Jeopardy about 15-20 minutes. There are intentionally more problems than can be played in a class period. The object of the game is to have fun reviewing and sorting the problem types, so there is some flexibility in the way you use the Jeopardy PowerPoint with your class. Note that students may be resistant to merely sorting problems and not solving them. This activity is about sorting, and solving practice will be done later in the lesson, in Final Jeopardy.)

Sorting Game

(See Jeopardy PowerPoint)

Our time is up for our round of Jeopardy. Now we will get ready for our round of FINAL JEOPARDY! *(Note: This activity is about 15 minutes.)* During Jeopardy, you did a very good job of sorting these problems into ratio, proportion, percent, and percent of change categories. When you worked on these problem types, you learned to use diagrams that helped you to represent and solve them. Sometimes, you will be asked to solve problems but will not have these diagrams available. We used the diagrams to represent information in the problem and to understand the relationships between different elements in the problems. When you solve problems, you do not have to use the exact diagrams we used in class to set up the problems. Instead, you can use your own diagrams that are efficient in representing information in the problem and useful in solving the different problems. This is what we are going to do in FINAL JEOPARDY.

Rules for FINAL JEOPARDY: First you will see a problem and decide the problem type, just like you did in the first round of Jeopardy. Next, you will work together to represent the problem. *(Note: State something about coming up with a diagram that is useful and quick to do.)* There may be different ways to set-up the information in a problem that you will share with the class. Let's look at one problem together.

(Display Example 19.2)

Example 19.2	The ratio of A's to B's that Sheetal received on her report card was 3:1. If Sheetal received 2 B's, then how many A's did she receive?

Teacher: What is the problem type?

Students: It is a ratio problem?

Teacher: How do you say this in "Jeopardy talk"?

Students: What is a ratio problem?

Teacher: Right. Now to earn your FINAL JEOPARDY points, you need to set up the problem using your own diagram/representation that would help you to solve the problem, and then solve the problem.

(Note: Walk through this problem with the students with the purpose of showing the students how to set up this problem without a teacher-directed diagram. It is recommended to play Final Jeopardy for about 15 minutes.)

A good question to ask yourself is, "What is the problem asking me to solve?" **So, tell me what do you have to solve in this problem?**

Students: We have to solve for the number of A's Sheetal earned on her report card.

Teacher: Excellent. Work with your partner to show how you would represent the problem and then solve it.

(Note: Circulate among the student groups giving guidance as necessary. Their work should include the use of correct units to align with the appropriate quantities. The following is an example:

$$\uparrow \frac{x \text{ A's}}{2 \text{ B's}} = \frac{3\text{A's}}{1 \text{ B}} \uparrow$$

(If students are working slowly or you are pressed for time, it is not necessary to have students solve this and other Final Jeopardy problems - you can instead ask that students merely set-up the problem and not solve it.) After most students have represented the problem, call on students to share their work. Discuss how each student example does or does not accurately represent information in the problem. Remind students that they need to have a good understanding of the relationships in the problem to solve it correctly, but that they do not need to use the exact diagrams used in instruction.)

(Display Final Jeopardy problems)

Problem 19.3	An official U.S. flag has a length-to-width ratio of 19:10. The U.S. flags at Martha Washington Elementary School measure 380 ft. by 210 ft. Are the U.S. flags at Martha Washington Elementary School official U.S. flags?

Problem 19.3: Answer

Problem type: What is a ratio problem?

Possible student representation:

$$\frac{19}{10} = ? \neq \frac{380}{210}$$

Answer: **No.**

Problem 19.4	Jihoon earns 10% more removing snow for people in his neighborhood than Sam earns in his neighborhood. If Sam earns \$20 for each driveway he clears of snow, how much does Jihoon earn?

Problem 19.4: Answer

Problem type: What is a percent of change problem?

Possible student representation:

$$\frac{\$?}{\$20} = \frac{10}{100}$$

\$20 + \$? = \$*x* (i.e., total \$ Jihoon earns for each driveway he clears of snow)

Answer: **\$22 per driveway.**

Problem 19.5	A carpenter makes miniature replicas of the furniture in the White House. The scale model of a table that he made is 4 inches long. The full-size table is 36 inches long. What is the model's scale?

Problem 19.5: Answer

Problem type: What is a proportion problem?

Possible student representation:

$$\frac{\text{Model}}{\text{Table}} = \frac{4}{36} : \frac{1}{9}$$

Answer: **1:9.**

Problem 19.6	Angie went shopping for a new comforter for her bed. She found an entire set of bedding (sheets, pillowcases and comforter) on sale for 40% off. The original price was \$120. What is the sale price of the bedding?

Problem 19.6: Answer

Problem type: What is a percent of change problem?

Possible student representation:

$$\frac{?}{120} = \frac{40}{100}$$

120 - ? = \$*x*

Answer: **\$72.**

Homework: pp. 175-184

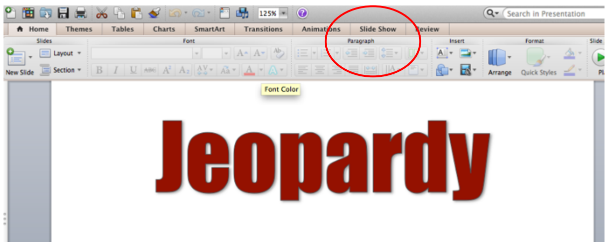
If students did not complete solving any of the problems that they represented during Final Jeopardy using their own diagrams in class, have them solve those problems for homework.

Jeopardy Sorting Game
Directions for Using the PowerPoint

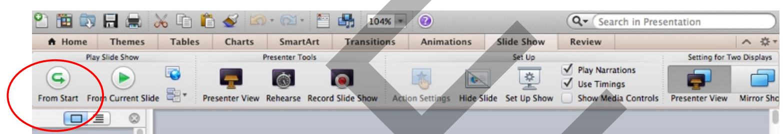
Regular Jeopardy

1. Open Jeopardy Sorting Game in PowerPoint.

2. Click slideshow tab.

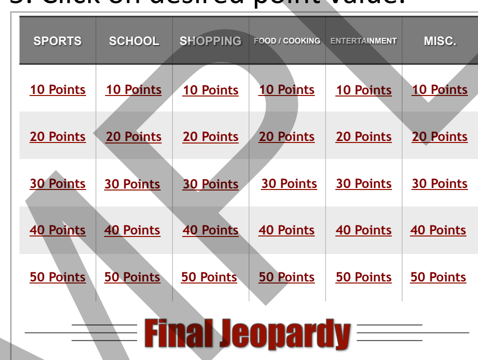


3. Click *From Start Icon.*

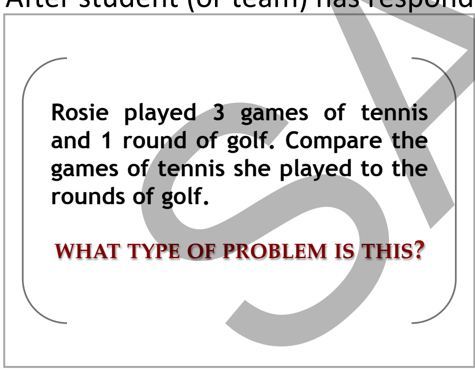


4. Click the screen to begin.

5. Click on desired point value.



6. After student (or team) has responded, click the screen to reveal the correct answer on the next slide.



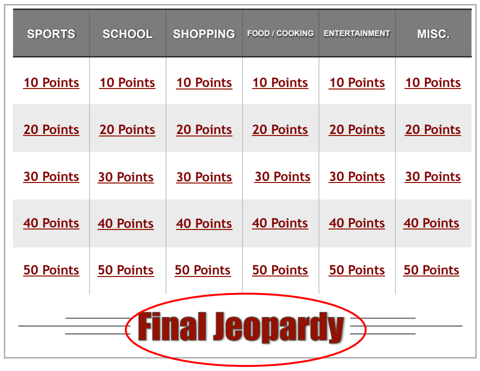
7. To return to category selection, click the screen with the correct answer. The PowerPoint will automatically return to the Category screen.

8. Click on desired point value for next problem. Repeat steps 5-7 to complete the Regular Jeopardy game.

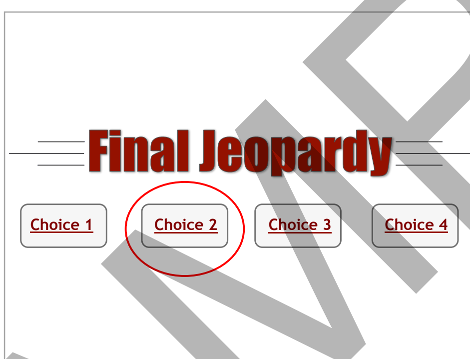
Final Jeopardy

After all questions have been answered, or time has run out, proceed to Final Jeopardy:

1. Click on the words "Final Jeopardy" on the bottom of the category selection screen:



2. Allow the students to choose from one of the four final Jeopardy choices. Click on the choice selected (e.g., click the words "Choice 2").



This will reveal the final Jeopardy question. Click the screen to reveal the answer. Clicking on the answer screen will bring you back to the title screen.

3. To end Jeopardy Game, press the escape button on keyboard.

Jeopardy Answers

		SPORTS	SCHOOL	SHOPPING	FOOD/COOKING	ENTERTAINMENT	MISC.
10 POINTS		Rosie played 3 games of tennis and 1 round of golf. Compare the games of tennis she played to the rounds of golf.	Terrence studied for 30 minutes and relaxed for 10 minutes. Compare the time Terrence studied to the time he relaxed.	Samantha went shopping and bought 3 shirts and 5 pairs of socks. Compare the number of shirts to pairs of socks.	Dominick wants to have enough wings so each boy can eat 6. There are 11 boys coming to the party. How many chicken wings should Dominick bring?	Their tickets are $60 each, and there is a 2% surcharge due to the ticket broker. What is the total cost for one ticket?	He has earned $7 per hour. This year, Max received a 10% increase in his pay. How much does he earn per hour now?
	Problem Type	Ratio	Ratio	Ratio	Proportion	Percent of Change	Percent of Change
	Answer	3:1	3:1	3:5	66 wings	$61.20	$7.70/hr
20 POINTS		Shayla usually plays 3 games of tennis for every round of golf. Last weekend she played 9 games of tennis. How many rounds of golf did she play?	Each classroom in a new middle school needs 7 chairs for every 2 tables. If Ms. Lincoln needs 4 tables, how many chairs will she need?	A store is selling Under Armor hats at 20% off their original price. What is the sale price of a hat originally priced at $20?	49% of students at school do not approve of the school's lunches. If there are 473 students who do not like the lunches, how many students are there in the school?	The bill with tax was $35.45, and Jenny and her friend want to give a 25% tip because they thought the service was excellent! What will be their total bill?	Maria's art gallery receives 30% commission on the art pieces it sells. If the gallery sells an art piece for $670, how much commission will they earn?
	Problem Type	Proportion	Proportion	Percent of Change	Percent	Percent of Change	Percent
	Answer	3 games of golf	14 chairs	$16.00	965 students	$44.31	$201.00
30 POINTS		The first lap of an auto race is 1000 meters. This is 15% of the total race distance. What is the total race distance?	A fundraiser raised $650 for a school trip. If 75% of the money raised was dedicated for the trip, how much money went toward the trip?	A shoe store marks up the price on Nike tennis shoes 45%. If the shoe store pays $40 for each pair of Nike's, what will be the price of shoes for customers at the store?	Rosa's recipe says to increase the cooking time by 4 % in high altitudes. If Rosa's recipe usually calls for 45 minutes of baking time, how long should she bake the cake in Denver?	Scott and Karen's meal costs $16.18. They want to leave a tip of about 20%. What is the amount of tip they should leave?	Noah runs about 8 miles an hour. Maintaining this speed, how long will it take him to run a 13 mile race?
	Problem Type	Percent	Percent	Percent of Change	Percent of Change	Percent	Proportion
	Answer	$6,666.67	$487.50	$58.00	47 min. *(46.8)*	$3.24	1.625 hours; 1 hour and 37.5 minutes

Jeopardy Answers

	SPORTS	SCHOOL	SHOPPING	FOOD/COOKING	ENTERTAINMENT	MISC.
40 POINTS	Mike's basketball team won 87% of the games they played over the past 2 seasons. If they played 58 games, how many games did they win?	The cheerleaders ordered new equipment that cost $382 plus a 4% delivery charge. What was the total bill to the cheerleaders?	Carissa spent 63% of her time at Abercrombie and Fitch. If she was at the mall for 5 hours, how much time did she spend at A and F?	Suppose Lucy's dog eats 2 lb of dog food every 3 days. How many pounds of food will the dog eat in 31 days?	You and a friend waited 40 minutes to get on the roller coaster at Valley Fair. The ride lasts 2 minutes. Compare the time you waited in line to the time you were on the ride.	A band has 50 members. Twelve members are percussionists. Compare the number of percussionists to total band members.
Problem Type	Percent	Percent of Change	Percent	Proportion	Ratio	Ratio
Answer	50 games *(50.46)*	$397.28	3.15 hours; 3 hrs and 9 mins	20.67 pounds	20 min:1 min	6:25
50 POINTS	Last year Val had an average bowling score of 240. This year her average increased 13%. What is her new bowling average?	Aaron received 320 points out of a possible 400 in his history class. What percent of the possible points did Aaron receive?	There was a deal for 3 boxes of General Mills cereals for $6.26. Sheryl bought 6 boxes of cereal for her family. How much did she spend?	A recipe for Rice Krispie squares asks for 6 cups of Rice Krispies and 2 cups of marshmallows. Compare the amount of marshmallows to Rice Krispies.	In his collection, Ben has 5 action DVDs for every 2 musical DVDs. If Ben has 15 action DVDs, how many musical DVDs does he have?	It rained 2 out of 5 days in the month of October. On how many days did it rain in the month?
Problem Type	Percent of Change	Percent	Proportion	Ratio	Proportion	Proportion
Answer	271.2	80%	$12.52	1:3	6 musical DVDs	12.4 days

FINAL JEOPARDY				
	An official U.S. flag has a length-to-width ratio of 19:10. The U.S. flags at Martha Washington Elementary School measure 380 ft by 210 ft. Are the flags at Martha Washington Elementary official U.S. flags?	Jihoon earns 10% more removing snow for people in his neighborhood than Sam earns. If Sam earns $20 for each driveway he clears of snow, how much does Jihoon earn?	A carpenter makes miniature replicas of the furniture in the white house. The scale model of a table that he made is 4 inches long. The full-size table is 36 inches long. What is the model's scale?	Angie went shopping for a new comforter for her bed. She found an entire set of bedding (sheets, pillowcases and comforter) on sale for 40% off. The original price was $120. What is the sale price of the bedding?
Problem Type	Ratio	Percent	Proportion	Percent of Change
Answer	No, because 19:10 = 1.9 and 380:210 = 1.81	$22.00	1 in : 9 inches	$72.00

RATIO, *PROPORTION*, and *PERCENT*
PROBLEM CHECKLISTS

Step 1: Discover the problem type

- Read and retell the problem to understand it.
- Ask if the problem is a …

a : b **RATIO PROBLEM**	*If…Then* **PROPORTION PROBLEM**	% **PERCENT, PERCENT OF CHANGE, or SIMPLE INTEREST PROBLEM**
Does this problem have a part-to-part or part-to-whole comparison? Look for symbols, words, and phrases such as: "the ratio of ***a*** to ***b***," "***a*** : ***b***," "***a*** per ***b***," "***a*** for ***b***," "***a*** for every ***b***," "for every ***b*** there are ***a***," "***n*** times as many/much as," "***n***th of," "***a*** out of ***b***," to see whether there is a **ratio statement** that tells about a multiplicative relationship between two quantities in a single situation.	Does the problem describe an "*If…Then*" statement of equality between two ratios/rates that allows us to think about the ways that two situations are the same? That is, the ***If*** statement describes a rate/ratio between two quantities in one situation and the ***Then*** statement involves either an increase or decrease in the two quantities in another situation, but with the same ratio.	Look for symbols or words such as "%," "percent," "percent of change," or "simple interest," to see whether there is a **percent or percent of change statement** that tells about a multiplicative relationship between two quantities.

- Ask if this problem is different from or similar to another problem that has already been solved.

Step 2: Identify information in the problem to represent in a diagram(s)

- Underline the …

ratio or comparison statement.	two quantities that form a specific ratio/rate.	percent or simple interest statement.

- Write …

➔ compared and base quantities and units in the diagram. ➔ value of the ratio between the two quantities in the diagram (**ratio value**). ➔ "***x***" for what must be solved.	➔ names of the two quantities that form a ratio in the diagram. ➔ quantities and units for each of the two ratios/rates in the diagram. ➔ "***x***" for what must be solved.	➔ information (part, whole, or ratio value; change, original, ratio value, or new) in the problem in the diagram(s). ➔ "***x***" for what must be solved.

Step 3: Solve the problem

- Try to come up with an estimate for the answer.
- Translate the information in the diagram into a math equation.
- Plan how to solve the math equation.
- Solve the math equation, and write the complete answer.

Step 4: Check the solution

- Look back to see if the estimate in Step 3 is close to the exact answer.
- Check to see if the answer makes sense.